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# Variability of the Electric Field Strength in the Earth-Ionosphere Waveguide Due to Variations in the Electron Density Profile

C. H. Shellman

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C. H. Shellman

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**ADMINISTRATIVE INFORMATION**

The work reported here was conducted for SPAWAR from 1 July 1990 to 31 December 1991.

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The author wishes to thank Richard A. Pappert, who suggested the problem addressed in this report.

## **EXECUTIVE SUMMARY**

### **PROBLEM**

A method is needed for finding the variability of the electric field strength in the earth-ionosphere waveguide at very low frequencies due to variations in the electron density profile.

### **RESULTS**

A method is developed for finding the standard deviation of the field strength in the earth-ionosphere waveguide. This method uses derivatives of the waveguide eigenangles with respect to height and slope variations in the electron density profile. The calculated results are in reasonable agreement with available data.

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## INTRODUCTION

Predictions of the field strengths of signals from very low frequency (VLF) and low frequency (LF) transmitters are of considerable interest to the Navy. For example, the choice of frequency diversity depends on the locations of nulls in the signal level. The predictions of field strengths are based on mode theory for the earth-ionosphere waveguide [Budden, 1961; Pappert, Gossard, and Rothmuller, 1967; Morfitt and Shellman, 1976]. Variations occur in the electron density profile, however, and a method of routinely calculating the standard deviation of the signal strength as a function of distance, due to these variations, has not been available. Such a method is needed for more realistic assessments of signal coverage.

In this report an exponential form of the electron density is assumed. The overall height is given by  $h'$  and the slope by  $\beta$  such that

$$N = 1.43 \times 10^7 \exp[\beta(h - h') - 0.15h]$$

where height is in kilometers and electron density,  $N$ , is in electrons per cubic centimeter. The parameters  $h'$  and  $\beta$  are assumed to have Gaussian distributions with standard deviations  $\sigma_h$  and  $\sigma_\beta$  with respect to time variations in the ionosphere.

Ferguson, Morfitt, and Hansen [1985] used measurements of signal levels of transmissions from NGR in Greece at 59 kHz to deduce  $h'$  and  $\beta$ , their standard deviations,  $\sigma_h$  and  $\sigma_\beta$ , and the correlation between them. The measurements were made at two locations on either side of a modal interference null at 900 km and 1300 km west of the transmitter. The object of this present work is to derive an efficient way of calculating the standard deviation of the field strength as a function of distance for any given values of  $\sigma_h$  and  $\sigma_\beta$ . It is assumed that there is no correlation between  $h'$  and  $\beta$ . This latter assumption is not fundamental to the formulation, however, which could be expanded to include correlation.

The approach taken in this present work is to find the rate of change of the sines of the waveguide eigenangles and from that information to find the expectation values of the field strength and the standard deviation. The excitation factors are assumed not to vary.

## THEORY

The expectation of the variance is

$$\begin{aligned} & \langle (E - \langle E \rangle) (E^* - \langle E^* \rangle) \rangle \\ &= \langle EE^* \rangle - \langle E \rangle \langle E^* \rangle - \langle \langle E \rangle E^* \rangle + \langle E \rangle \langle E^* \rangle \\ &= \langle EE^* \rangle - \langle E \rangle \langle E^* \rangle \end{aligned}$$

where the electric field,  $E$ , as a function of  $h'$  and  $\beta$ , is taken to be

$$E = \sum \lambda_n \exp \{ -ik[S_n + (\partial S_n / \partial h') \Delta h' + (\partial S_n / \partial \beta) \Delta \beta]x \}$$

where  $\lambda_n$  is the excitation factor,  $S_n$  is the sine of the earth-ionosphere eigenangle, and  $k$  is the wave number. The variation with respect to  $h'$  and  $\beta$  is taken to be independent.

In general

$$\langle f(w) \rangle = \int f(w) p_1(w) dw$$

where the integral is taken from  $-\infty$  to  $\infty$  and

$$p_1(w) = [1/\sigma(2\pi)^{1/2}] \exp(-w^2/2\sigma^2)$$

For a sum

$$\begin{aligned} \langle \sum a_n(w) \rangle &= \int [\sum a_n(w)] p_1(w) dw \\ &= \sum \int a_n(w) p_1(w) dw = \sum \langle a_n(w) \rangle \end{aligned}$$

For  $a_n$  of the form

$$a_n = \exp(-a_0 - a_1 w)$$

the expectation is

$$\langle a_n \rangle = [1/\sigma(2\pi)^{1/2}] \int \exp(-a_0 - a_1 w - w^2/2\sigma^2) dw$$

where the subscript  $n$  has been omitted from the constants  $a_0$  and  $a_1$  for simplicity.

Completing the square

$$a_1 w + w^2/2\sigma^2 = (a_1\sigma^2 + w)^2/2\sigma^2 - a_1^2\sigma^2/2$$

so that

$$\langle a_n \rangle = [1/\sigma(2\pi)^{1/2}] \exp(-a_0 + a_1^2\sigma^2/2) \int \exp[-(a_1\sigma^2 + w)^2/2\sigma^2] dw$$

The integral is equal to  $\sigma(2\pi)^{1/2}$  so that

$$\langle a_n \rangle = \exp(-a_0 + a_1^2\sigma^2/2)$$

The result is easily extended to the case of two independent variables,  $w_a$  and  $w_b$ ; that is, for

$$a_n = \exp(-a_0 - a_1 w_a - b_1 w_b)$$

The expectation is then of the form

$$\langle a_n \rangle = \int \int a_n(w_a, w_b) p_2(w_a, w_b) dw_a dw_b$$

where

$$p_2(w_a, w_b) = (1/2\pi \sigma_a \sigma_b) \exp[-(w_a^2/\sigma_a^2 + w_b^2/\sigma_b^2)/2]$$

so that

$$\langle a_n \rangle = \exp(-a_0 + a_1^2 \sigma_a^2/2 + b_1^2 \sigma_b^2/2)$$

The expectation of  $E$  is then proportional to

$$\langle E \rangle \sim \sum \lambda_n \exp[-ik S_n x - (kx \sigma_h \partial S_n / \partial h')^2/2 - (kx \sigma_\beta \partial S_n / \partial \beta)^2/2]$$

Note that the expectation of  $E$  is not the value of  $E$  for  $\Delta h' = \Delta \beta = 0$ . This is because of the asymmetrical nature of the exponential function.

The expectation of  $EE^*$  is proportional to

$$\begin{aligned} \langle EE^* \rangle \sim \sum_n \sum_m \lambda_n \lambda_m^* \exp\{-ik(S_n - S_m^*)x - [kx \sigma_h (\partial S_n / \partial h' - \partial S_m^* / \partial h')]^2/2 \\ - [kx \sigma_\beta (\partial S_n / \partial \beta - \partial S_m^* / \partial \beta)]^2/2\} \end{aligned}$$

The derivatives of sine  $\theta$  with respect to  $h'$  and  $\beta$  were computed by finding eigenangles for finite increments in  $h'$  and  $\beta$ . The independence of  $h'$  and  $\beta$  is assumed. The derivatives are then

$$\partial S_i / \partial h' = \cos \theta \{ [\theta_i(h' + \Delta h') - \theta_i(h' - \Delta h')] / 2\Delta h' \}$$

$$\partial S_i / \partial \beta = \cos \theta \{ [\theta_i(\beta + \Delta \beta) - \theta_i(\beta - \Delta \beta)] / 2\Delta \beta \}$$

Only the 10 or 12 most significant modes were included. Care was taken to ensure that the same set of modes was used for the increments and decrements in  $h'$  and  $\beta$  as for the no-increment set. Increments  $\Delta h' = 1\text{km}$  and  $\Delta \beta = 0.1$  were used for this purpose.

In principle, analytic derivatives of the eigenangles could be computed so that ordering would not be a problem. Although computer code for this is already available, it is large and would unduly complicate the MODESRCH program.

## RESULTS

Calculations of the variability of electric field strength in the earth-ionosphere waveguide were compared with data taken on board a ship in the Atlantic, the



GTS *Callaghan*, from which GBR at Rugby, England; the Anthorn transmitter in England; NSS at Annapolis; and NAA at Cutler, Maine, were monitored [Computer Sciences Corporation, 1989]. The data were taken at night on many traverses of the Atlantic, and hence the superimposition of these data gives a good indication of the nighttime variability in the  $E$ -field due to variation in the ionosphere.

In the calculations, each path was approximated to be entirely over seawater even though in fact a small part of each path in the vicinity of the transmitter was over land. Each path was approximated to be homogeneous in other respects as well. Actually the azimuth of propagation and the dip and strength of the earth's magnetic field varied somewhat along each path. For seawater, the relative permittivity  $\epsilon_r = 81$ , and the conductivity  $\sigma = 4.0$ . Twelve modes were used for comparisons with NSS data and ten modes were used for each of the other cases.

Figures 1a, 2a, 3a, and 4a show a comparison between field strength data and field strength calculations for GBR (16 kHz), Anthorn (19 kHz), NSS (21.4 kHz), and NAA (24 kHz), respectively (figures are at the end of the report). The fit is fairly close, considering that homogeneity along the path is assumed.

Figures 1b, 2b, 3b, and 4b show the data with the expectation of the field strength as a function of distance. Curves of standard deviation are also shown. These are for  $\sigma_h = 1.0$  and  $\sigma_\beta = 0.05$ . It is seen that the magnitudes of the standard deviations for the computed curves are about the same as for the data.

The expectation of the field strength and the standard deviations are also shown in the remaining figures for various combinations of  $\sigma_h$  and  $\sigma_\beta$ . In figures 1c, 2c, 3c, and 4c,  $\sigma_h = 1.0$  and  $\sigma_\beta = 0.05$ , and the usual curve of field strength versus distance is shown as a dashed line.

In figures 1d, 2d, 3d, and 4d,  $\sigma_h = 1.0$  and  $\sigma_\beta = 0.0$ , and in figures 1e, 2e, 3e, and 4e,  $\sigma_h = 0.0$  and  $\sigma_\beta = 0.05$ . These last figures show the separate effects of variation in  $h'$  and  $\beta$ . The variation of  $\beta$  affects the field mostly at long distances. This would correspond to the fact that the slope of the electron density profile affects the attenuation much more than does the height.

## CONCLUSIONS

A method has been presented of finding the standard deviation of the field strength in the earth-ionosphere waveguide. This method uses derivatives of the eigenangles with respect to height and slope variations in the electron density profile. The calculated results are in reasonable agreement with available data.

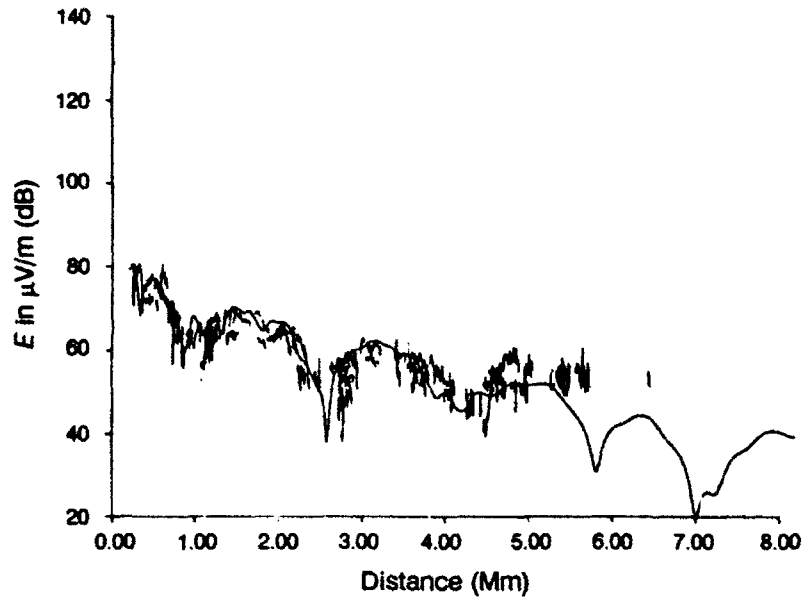
A method that uses the derivatives of the eigenangles is considered to be better than one that uses the derivatives of  $E$  or  $\ln|E|$  directly since the variation of the eigenangles is

more nearly linear. The curve of the expectation of  $\ln|E|$  is smoother than that of  $\ln|E|$ , and the expectation of the standard deviation of  $\ln|E|$  shows, not surprisingly, a considerable amount of asymmetry.

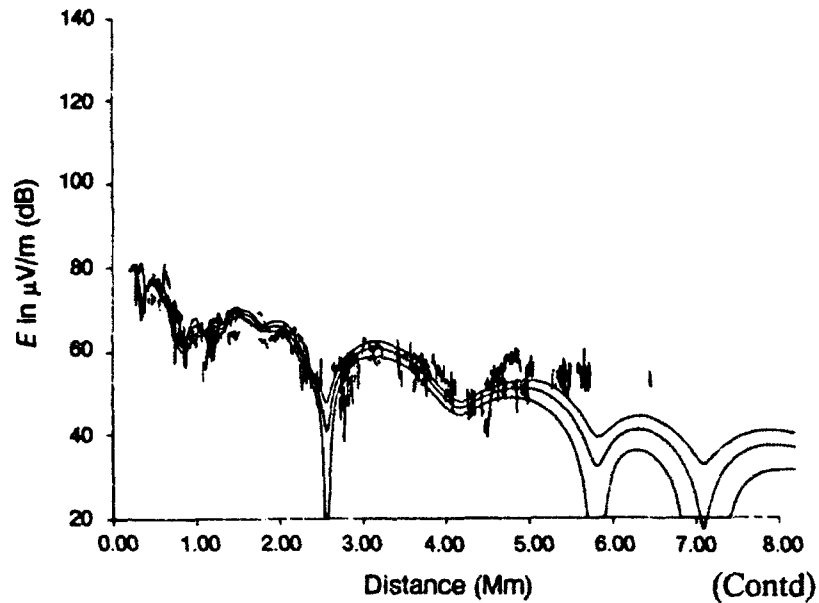
The method has some drawbacks. Sets of modes for the various profiles needed to find the derivatives must be visually checked for each case, and the situation is ambiguous when modes are close together. Variation of the excitation factors is not accounted for. Furthermore, the amount of computation is proportional to the square of the number of modes once the sets of modes are found. Nevertheless, the method is expected to be useful in predicting the variation of signal strength in the earth-ionosphere waveguide.

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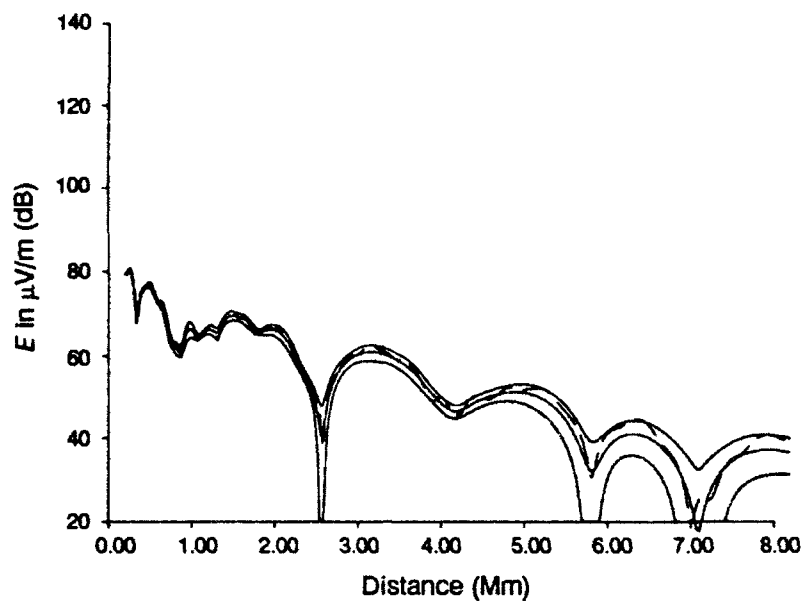


(a) Calculated field strength compared with GBR data.

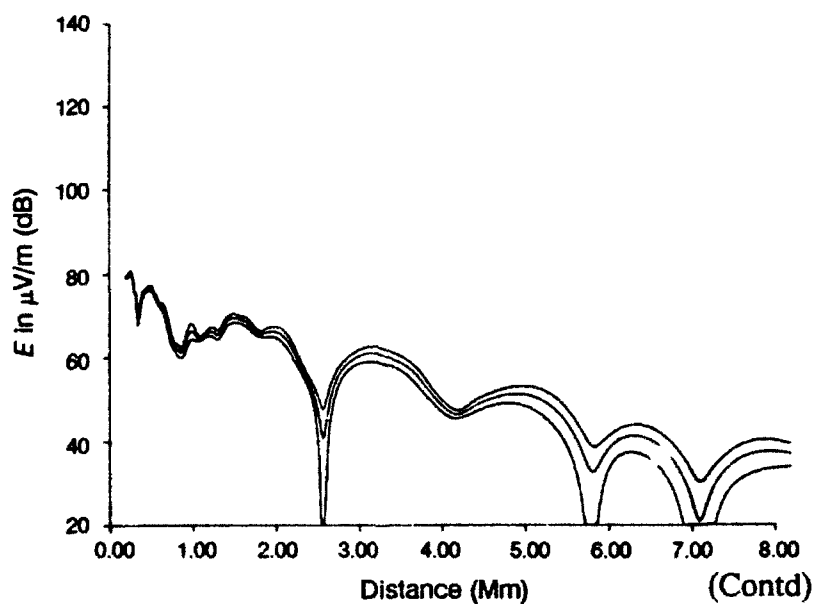


(b) Expectation of the field strength and standard deviation compared with GBR data.  $\sigma_h = 1.0 \text{ km}$  and  $\sigma_\beta = 0.05 \text{ km}^{-1}$ .

Figure 1. Comparison of calculated values with GBR data. Frequency = 16.0 kHz, azimuth = 265 degrees, dip = 66 degrees, and magnetic field strength = 0.47 gauss. Parameters of the profile are  $h' = 83.8 \text{ km}$  and  $\beta = 0.5 \text{ km}^{-1}$ .

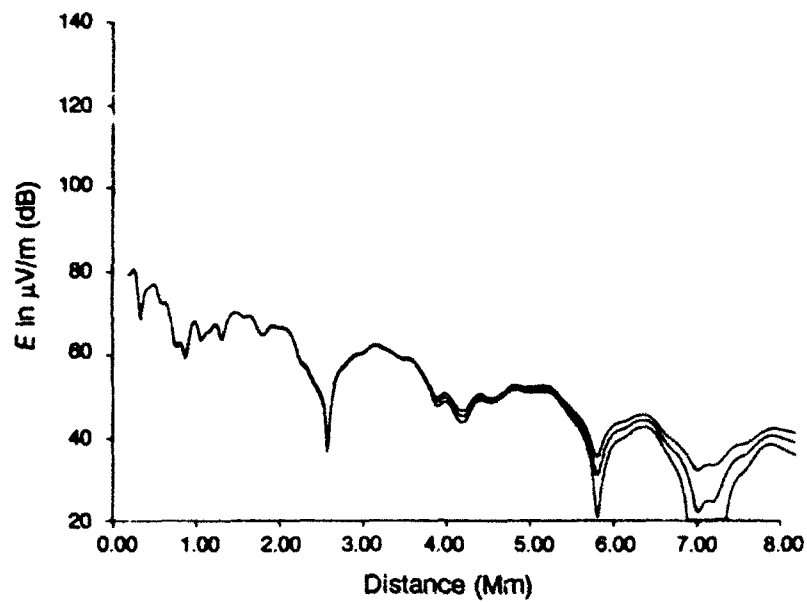


(c) Field strength (dashed curve), expectation of field strength, and standard deviation.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.05$  km<sup>-1</sup>.



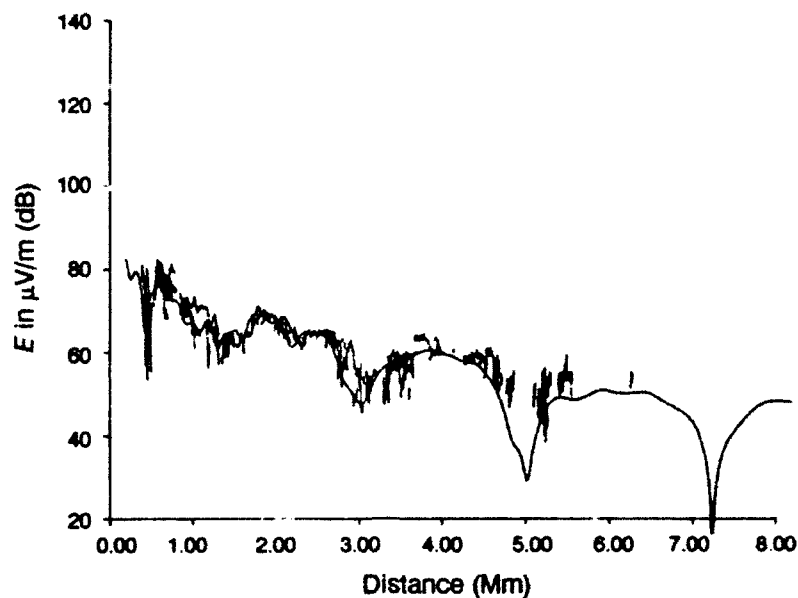
(d) Expectation of the field strength and standard deviation for variability of the height of the profile.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.0$ .

Figure 1. Continued.

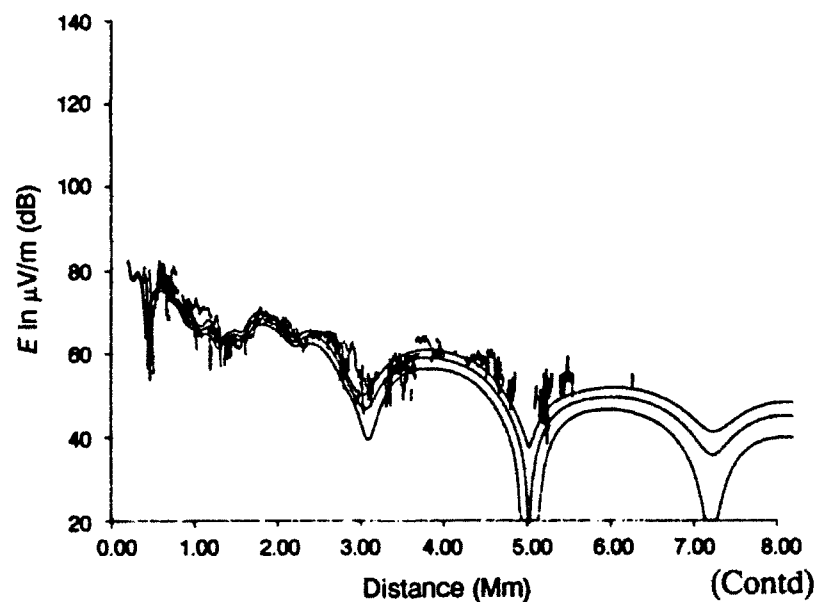


(e) Expectation of the field strength and standard deviation for variability of the slope of the profile.  $\sigma_h = 0.0$  and  $\sigma_\beta = 0.05 \text{ km}^{-1}$ .

Figure 1. Continued.

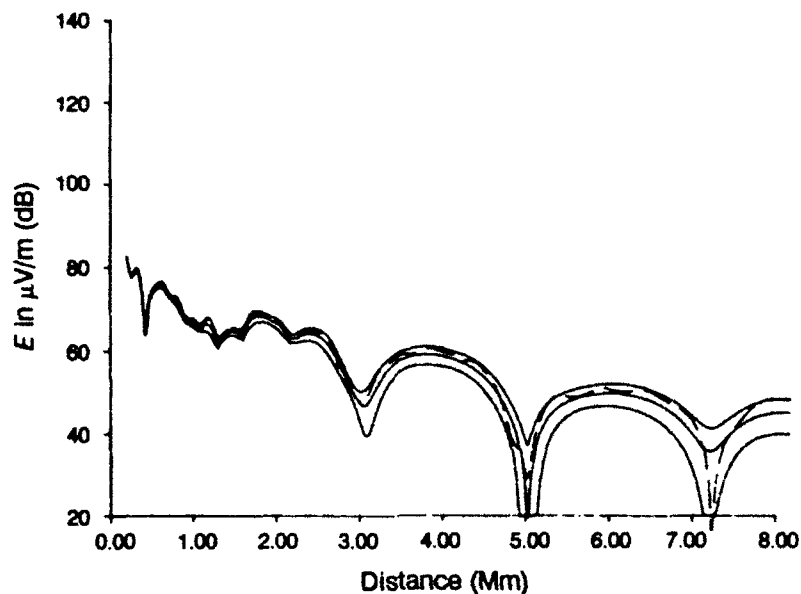


(a) Comparison of calculated values with Anthorn data.

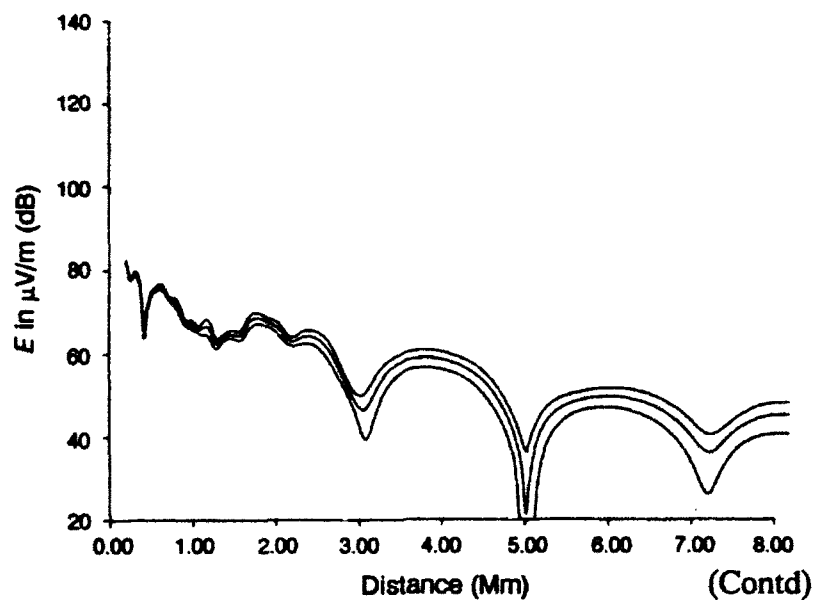


(b) Expectation of the field strength and standard deviation compared with Anthorn data.  $\sigma_h = 1.0 \text{ km}$  and  $\sigma_\beta = 0.05 \text{ km}^{-1}$ .

Figure 2. Comparison of calculated values with Anthorn data. Frequency = 19.0 kHz, azimuth = 265 degrees, dip = 69 degrees, and magnetic field strength = 0.48 gauss. Parameters of the profile are  $h' = 84.0 \text{ km}$  and  $\beta = 0.5 \text{ km}^{-1}$ .

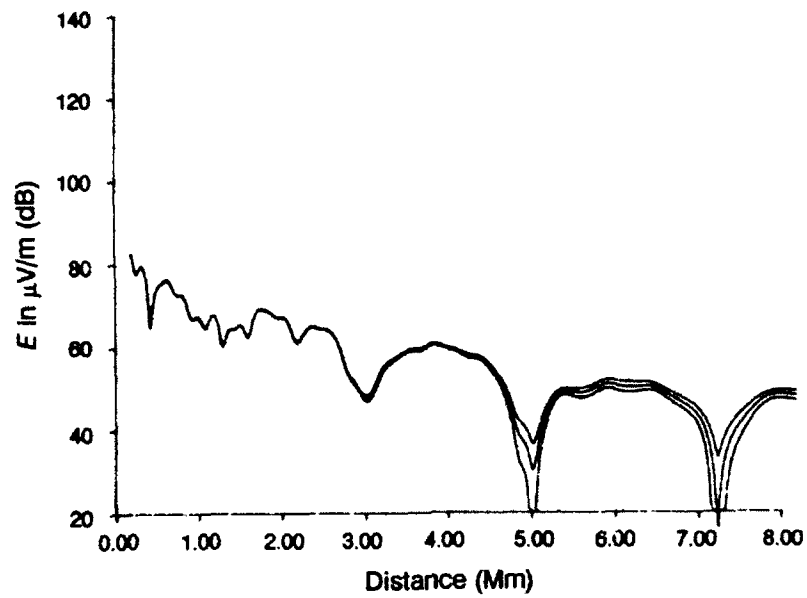


(c) Field strength (dashed curve), expectation of field strength, and standard deviation.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.05$  km<sup>-1</sup>.



(d) Expectation of the field strength and standard deviation for variability of the height of the profile.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.0$ .

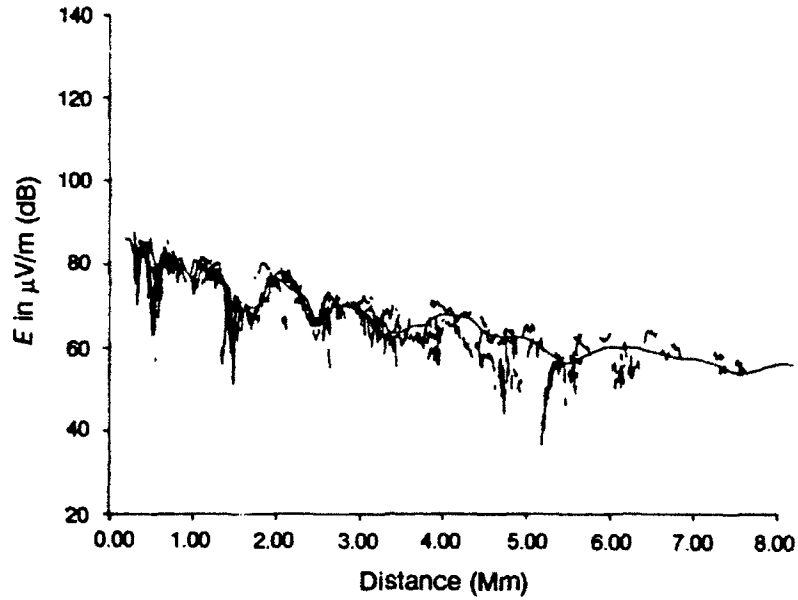
Figure 2. Continued.



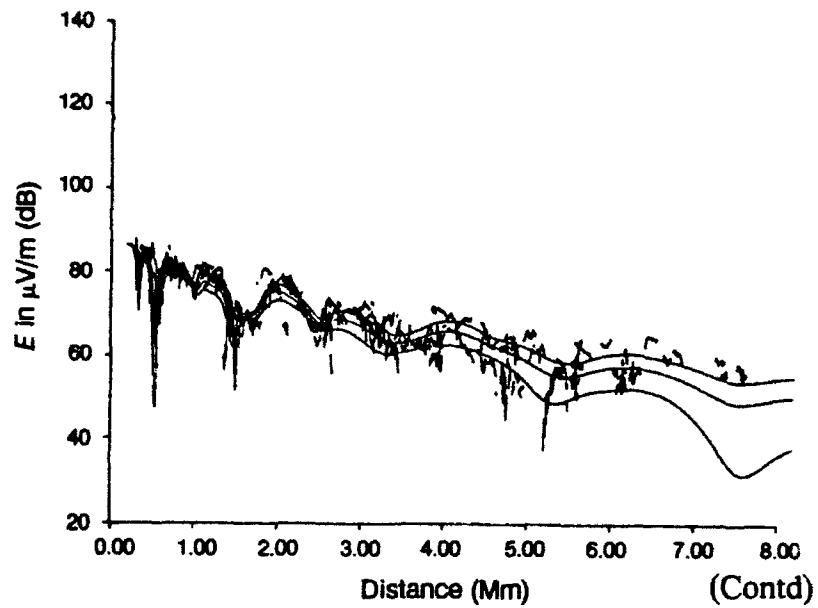
(e) Expectation of the field strength and standard deviation for variability of the slope of the profile.  $\sigma_h = 0.0$  and  $\sigma_\beta = 0.05 \text{ km}^{-1}$ .

Figure 2. Continued.



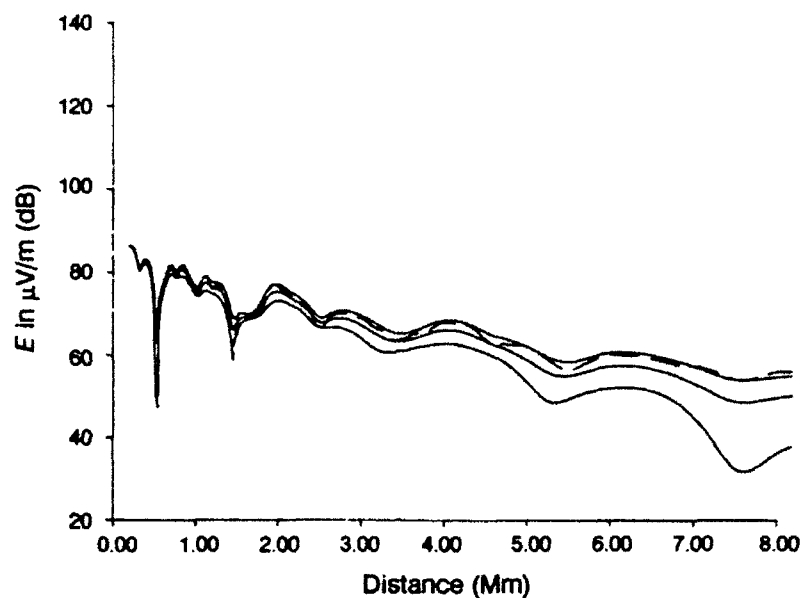


(a) Comparison of calculated values with NSS data.

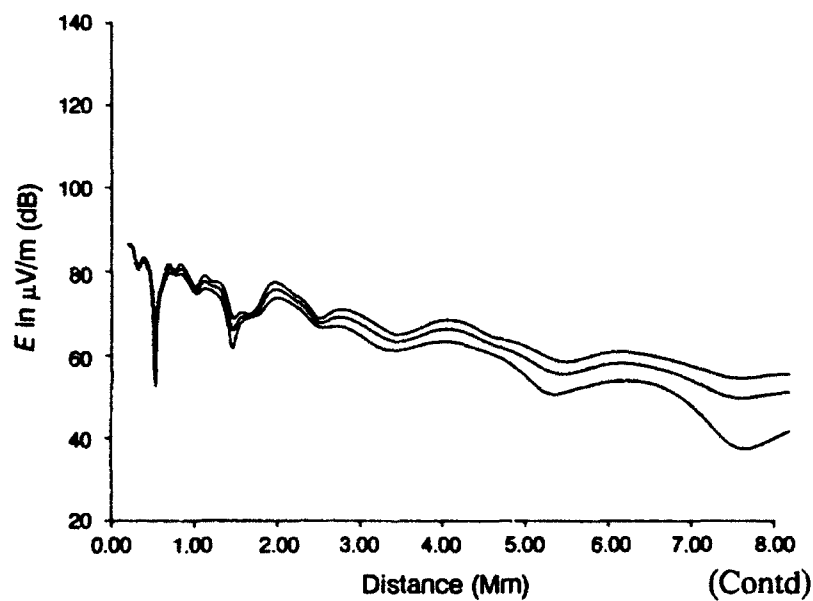


(b) Expectation of the field strength and standard deviation compared with NSS data.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.05$  km<sup>-1</sup>.

Figure 3. Comparison of calculated values with NSS data. Frequency = 21.4 kHz, azimuth = 85 degrees, dip = 72 degrees, and magnetic field strength = 0.53 gauss. Parameters of the profile are  $h' = 85.3$  km and  $\beta = 0.5$  km<sup>-1</sup>.

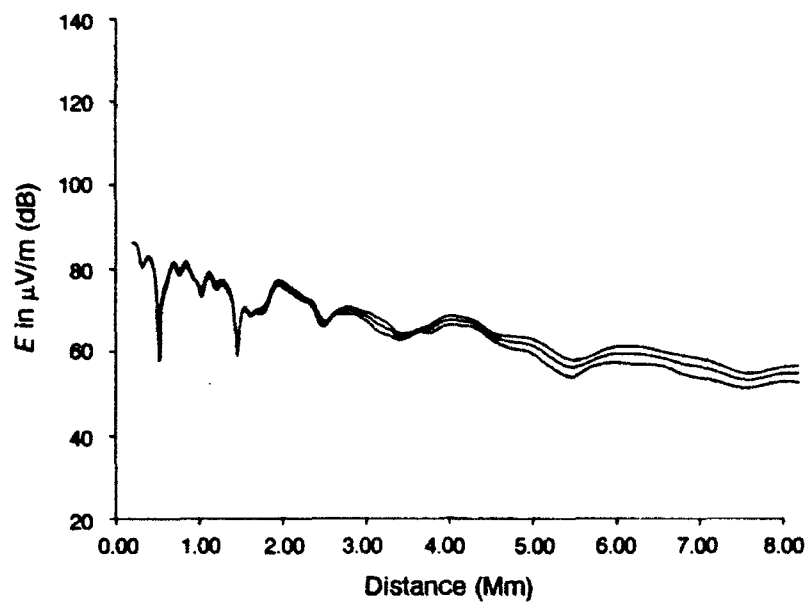


(c) Field strength (dashed curve), expectation of field strength, and standard deviation.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.05$  km<sup>-1</sup>.



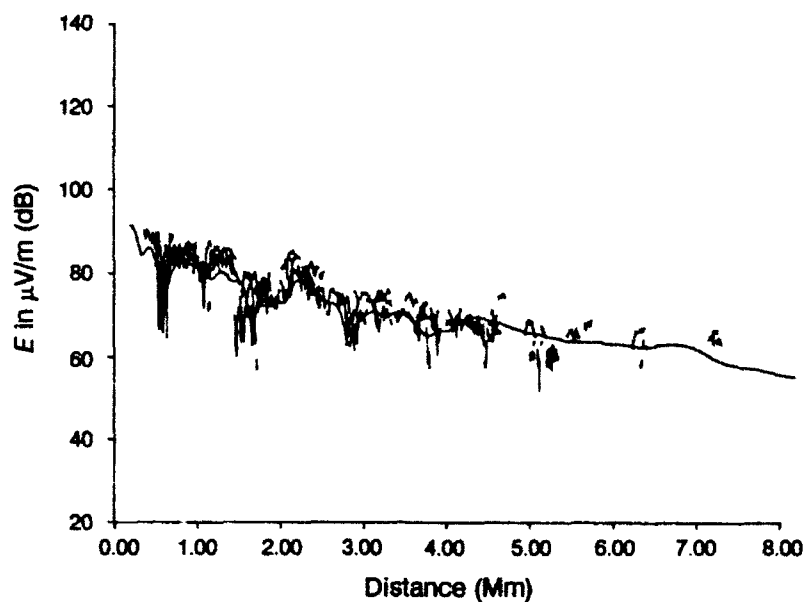
(d) Expectation of the field strength and standard deviation for variability of the height of the profile.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.0$ .

Figure 3. Continued.

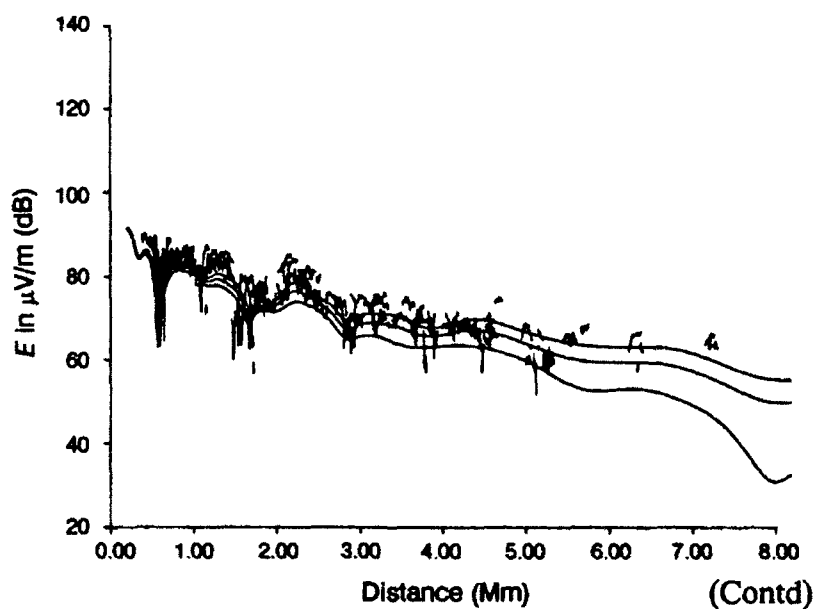


(e) Expectation of the field strength and standard deviation for variability of the slope of the profile.  $\sigma_h = 0.0$  and  $\sigma_\beta = 0.05 \text{ km}^{-1}$ .

Figure 3. Continued.

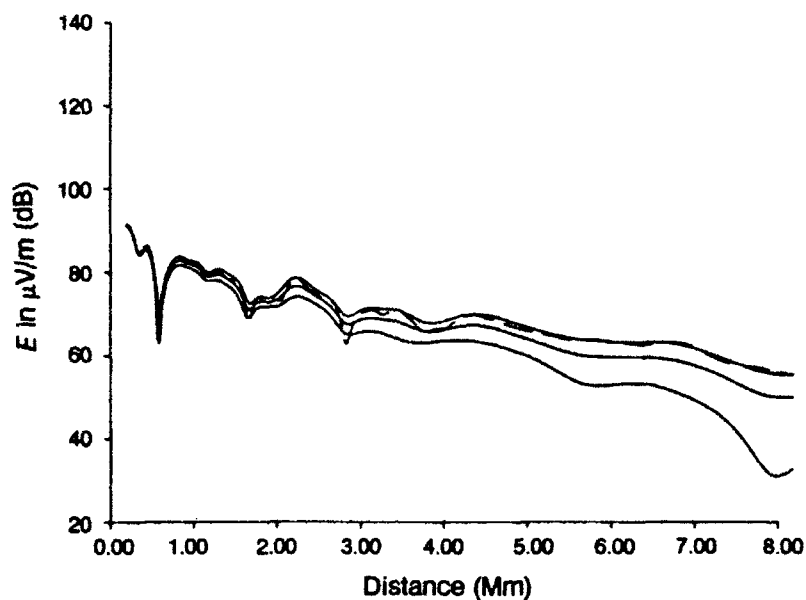


(a) Comparison of calculated values with NAA data.

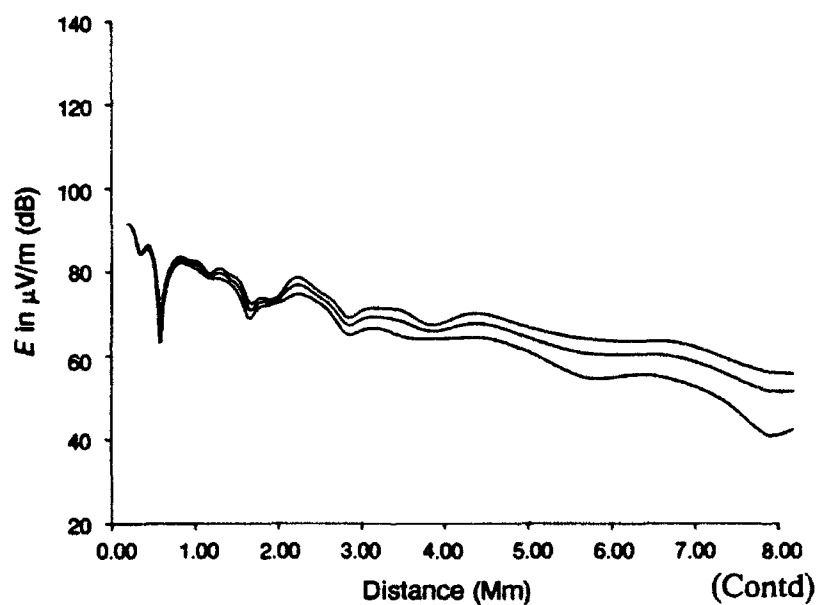


(b) Expectation of the field strength and standard deviation compared with NAA data.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.05$  km<sup>-1</sup>.

Figure 4. Comparison of calculated values with NAA data. Frequency = 24.0 kHz, azimuth = 120 degrees, dip = 70 degrees, and magnetic field strength = 0.50 gauss. Parameters of the profile are  $h' = 85.1$  km and  $\beta = 0.43$  km<sup>-1</sup>.

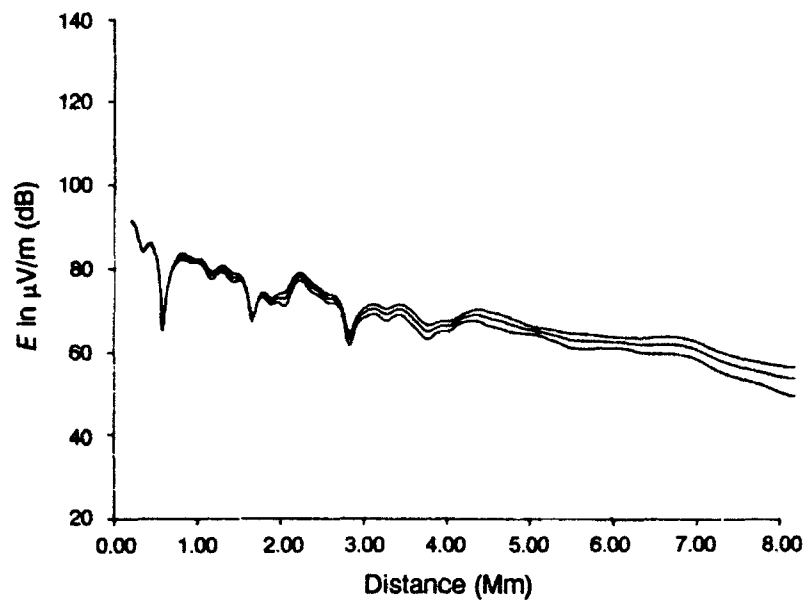


(c) Field strength (dashed curve), expectation of field strength, and standard deviation.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.05$  km<sup>-1</sup>.



(d) Expectation of the field strength and standard deviation for variability of the height of the profile.  $\sigma_h = 1.0$  km and  $\sigma_\beta = 0.0$ .

Figure 4. Continued.



(e) Expectation of the field strength and standard deviation for variability of the slope of the profile.  $\sigma_h = 0.0$  and  $\sigma_\beta = 0.05 \text{ km}^{-1}$ .

Figure 4. Continued.

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